A Numeral Toolbox

WRITTEN BY
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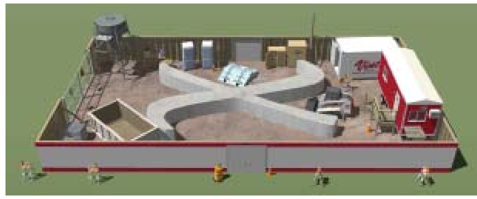


This article explores various strategies and tools which can be drawn upon to develop new numeral systems. Like the tools in a master carpenter's toolbox, these methods can help produce work; only the master carpenter's experience and planning can guide these tools to produce good work. One will need good criteria for the effective design of new numerals. This article is strictly concerned with tools.

Simple Graphic Tools

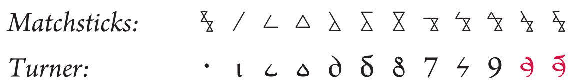
Figure 1: *Examples of four simple tools to derive new numerals from existing symbols.*

Presented below are simple graphic tools for deriving a new symbol from an antecedent. Rotation and reflection are usually pure wholesale transformations of a given existing symbol, while smoothing and justification are governed by intuition and the author's creativity. These tools apply to symbols derived from antecedent symbols in the public lexicon, or to invented symbols implied by a pure strategy.

Rotation and Reflection are simple transformations of everyday symbols to derive new symbols. These simple tools can be observed at work within the Hindu Arabic numeral set itself in the 6 and 9, and in the alphabet in the letters p, q, b, and d.

Elision and Appendage simply involve eliminating or introducing an element to a symbol to produce a new symbol. A. Chilton arrives at a "hatless" five (ɂ). Diacritical marks, such as those used by J. Halcro Johnson to build a "negative four" (4̄) out of a four (4), are common. F. Ruston uses an appended curl at the lower right of his numerals to signify that six has been added to their value (Π = 3, Π̨ = 9). In this article, we'll distinguish "free-floating" additions to a symbol as diacritical marks, whereas those additions which are attached to the numeral are appendages.

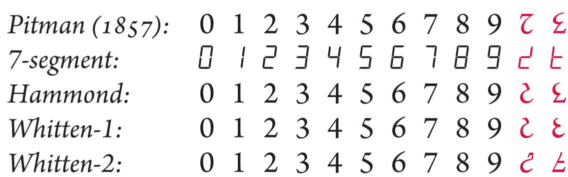
Smoothing simply involves adapting a symbol to everyday handwriting. The evolution of today's letters and numbers is an empirical process that takes place over generations; redundancies and hard angles are lost; differences between similar symbols tend to accentuate, further distinguishing them. Smoothing is an author's attempt to anticipate the evolution of figures. Gwenda Turner produced a set of symbols which derive from arrangements of matchsticks. The matchstick arrangements, Turner argues in *Dozenal Journal* 4 page 4, can be used to teach children the relationship of quantity to number. The numerals thus serve as an analog of the quantity they represent. Turner adds "If a dozen were the standard base it would be necessary to write the numerals at speed. For this, a cursive form is more efficient, and △ [her numeral three] would become ▵." Turner adds that the cursive, rounded triangle, perhaps taken as a circle, would force the zero be conveyed by "another symbol, such as the Arabic · [a raised dot]”. In the written form of her analog numerals, the complex of “matchsticks” that convey numeral six [⧖] can be bundled and represented as a leftward stroke at the bottom of the numerals larger than six:



Cultural Resonance. The producer of a new set of numerals may be inclined to force a symbol produced by a pure strategy to conform to a familiar configuration. This is seen mostly in “least change” strategies where the existing “Hindu-Arabic” numerals are retained, as most people are already familiar with their value.

Justification. An existing symbol, like the numeral “4”, can be forced to comply with a technical constraint. This may include compliance with a grid as in a dot-matrix screen or a tiling layout. The 7 or 13 segment LCD/LED digital readout is a common constraint.

Don Hammond [1], honorary DSA Member active in the DSGB, and Niles Whitten [2] modified Sir Issac Pitman’s 1857 transdecimals to make these more compliant with the 7 segment LCD/LED digital readouts. (See Vol. 4↊; № 2 page 17;):



Strategy. The following techniques relate more to a governing methodology rather than sheer graphic manipulation. We’ll back up the concepts with examples. Strategies can be “pure”, meaning they are allowed to function as ideas without intervention by the author. The author can also interrupt a strategy to serve other strategies: multiple strategies may be at work. Some ideas are permitted to run over a sequence of numerals, then are interrupted: this sequence we’ll call a *run sequence*. Some authors smooth or justify the “pure output” of a given strategy. It might be clearly observed by the reader that some “pure” strategies yield a set of symbols which are difficult to distinguish from one another when seen in mass. This tends to serve as impetus to distinguish some numerals from others, sacrificing purity for legibility. No judgments will be made regarding an author’s application of strategy; we’ll simply observe which strategy appears to be at work.

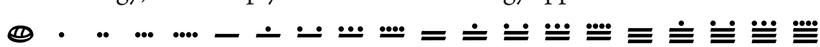


Figure 2: *Mayan numerals zero through one dozen seven.*

The Analog Strategies. The simplest means of developing graphic symbols intended to convey numerals is by the use of an *analog*, or a representation of a real-world concept through the use of a symbol which more or less directly resembles that concept itself. A clear example of this occurs among the Mayan numerals. One dot represents the number one, two dots represent “two”, etc. The Mayan numerals one through four constitute the basic run sequence for Mayan numerals. This type of basic run sequence, wherein each element can be counted and

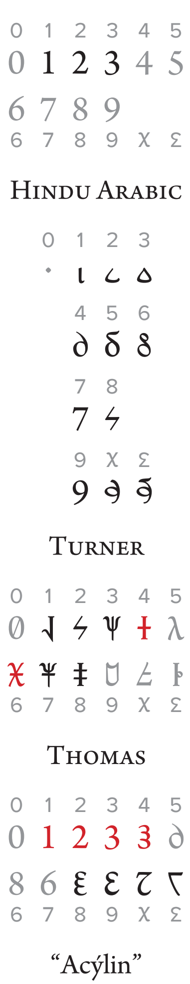


Figure 3: *Studies of the apparent incomplete or interrupted additive analog run sequences of several separate identity symbologies. The small gray numerals above or below the symbols are the “standard” duodecimal digits which the symbols convey.*

yield the digit the numeral represents we will call an *additive analog*. At number five, the Mayans introduce an element intended to convey the idea of a grouping we'll call a *bundle*. In this case, the bundle is a “bar” that signifies a group of five. As we count to nine, the basic run sequence set above the bundle generates the numerals six through nine. At ten, the Mayans stack their bundles, one atop the other, “two fives”. The bundles stack again at one dozen three. Thus the numerals five, ten, and one dozen three represent a bundle sequence. This bundle sequence can be called a *multiplicative analog*. The “standard” Mayan numeral set is a relatively pure system strategically. The numeral zero symbolizes emptiness; its numeral is symbolic. Zero seems to be symbolic in every system; it's difficult to be literal about the numeral zero without puzzling spaces cropping up to confuse the reader.

We've already examined the run sequences and bundles apparent in Gwenda Turner's “matchsticks” and smoothed numerals. It's interesting to observe the handwritten numerals to see that a different set of run sequences become apparent. (See Figure 3).

Roman numerals are also analogous in the first three numerals: I is one stroke and means “one”, II is two strokes and means “two”, etc. Beyond III, in most cases (but not in the case of clock faces), the numeral is generated using a different, more complex concept. The first three Roman numerals constitute the basic run sequence for Roman numerals. (The Roman numeral system is not a positional-notation system and really can't be compared meaningfully with dozenal symbologies recently proposed, except for Brother Louis Francis' “Roman Dumerals” described in Vol. 10; № 1 page 7;, which are beyond the scope of this article).

The Hindu Arabic numerals are somewhat analogous in the first three nonzero numerals; the system having undergone evolution from earlier forms which were perhaps even more clearly analogs. Let's look at analogs among dozenal symbologies.

Additive Analog Strategy. Rafael Marino, professor at Nassau Community College in New York, described a system in Vol. 38; № 2 page 10; wherein a set of numerals are devised and function similar to the Mayan system. The numerals in the basic run sequence, numerals {1, 2, 3, 4, 5}, are defined simply by producing a number of strokes, one above the other, in analogy to the number the numeral represents. At {6}, a vertical bar bundles 6; thereafter, appending the bundle element to the basic run sequence supplies {7, 8, 9, Χ, Ɛ}. Zero is symbolized by a box.

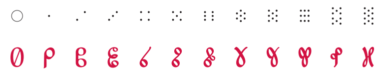
F. Ruston's numerals, conveyed in 1961 by the DSGB's *Dozenal Newscast*, Year 3, № 2, page 3, establishes its basic run sequence {1, 2, 3, 4, 5} using segments in the numerals. The numeral corresponding to 5 (Ⅹ) has five segments but can be efficiently produced in three strokes. At {6}, Ruston introduces a "curlicue" bundle, which thereafter can be appended to the lower right end of each of the basic run sequence numerals to obtain {7, 8, 9, Χ, Ɛ}.

P. D. Thomas, based in South Australia, produced a couple pamphlets revised in 1987 entitled "Modular Counting" and "The Modular System". The proposed symbology features number forms which are purely creative, except for the fact that for the digits {1, 2, 3, 4, 6}, the number of "appendages" equals the integer it represents. In this interrupted range, Thomas' numerals function as analogs. Some symbols, such as "dow" (ϟ), representing two, derive "from the mirror image of the [Hindu-Arabic] figure 2". Others, such as "okt" (ǂ), representing 8, "being twice kaw [Thomas' symbol for four], it has the vertical line with two horizontal arms," the four having four points, the symbol for eight "doubling" the symbol for four ideogrammatically, but ending up with six points. Some elements have to be ignored in order to construct the numeral Thomas intends. In order to construct the notion of "one" from Thomas' digit-one, ignore the "flag" at its lower left. The appendages to Thomas' two and three sprout from a horizontal bar and a vertical pole, respectively. Thomas writes that some of his numerals, for example, "vin" (λ), are "entirely arbitrary". Thus Mr. Thomas' symbology is a less-purely analogous system, with an interrupted and impure basic additive run sequence:

R. J. Hinton's proposal, published in the *Dozenal Newscast*, Year 3, № 2, page 7, were developed having "visually consider[ed] each [numeral] with the amount it represents". The basic run sequence {1, 2, 3} enumerates loops. A second run sequence {4, 5, 6} presumes 3 is added, and loops are again enumerated, the new loops being arranged in a distinct manner from the first sequence. The sequence {7, 8, 9} continues the same idea presuming 6 is added to the enumeration. Hinton's article conceptually employs a system of dots similar to those seen on dominos, around which he builds new numerals. The numerals for digit-ten and digit-eleven appear to be mildly related to the pattern of dots Hinton used to build his set [3]. Hinton's symbology basically runs an

Figure 4. *Studies of the additive analog run sequences of several separate identity symbologies.*

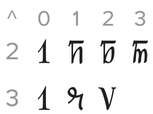
analog strategy in three groups of three, then abandons the strategy at digit ten:



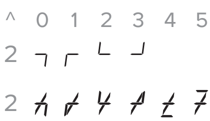
Fred Newhall, past DSA President, introduced his system of "efficient number symbols" in Vol. 2X; № 3 on page 16;. The system functions similar to shorthand and is unique in that it is truly cursive. The system is not analog but is arranged in a series of brief run sequences. Mr. Newhall generally arranged odd and even numerals in pairs, with the odd number reversed or repeated to obtain the next numeral. The digit-eleven perhaps supplies the symbol for zero through rotation:

Positional / Rotational Additive Analogs. A system of numerals can be produced by rotating one symbol to indicate the numeral's value, much like on a clock face. The purest manifestation of this idea follows. In the nineties, Prof. Bill Lauritzen devised a set of "gravity-generated numerals" in his paper "Nature's Numbers," retrieved at http://www.earth360.com/math-naturesnumbers.html. The numerals function as miniature six-position analog clocks with one hand that indicates the integer the figure represents. Numbers smaller than 6 have hands on the interior of a circle, those six or greater have hands outside the circle. Lauritzen's symbology is perhaps among the more purely geometrical and abstract:

"Arqam"



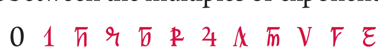
Ferguson



Schumacher-Zirkel

Figure 5. *Studies of the multiplicative strategy exhibited by the large "arqam" least change symbology, and exponential strategy for Shaun Ferguson's and the Schumacher and Zirkel-modified binary-coded analog separate identity sequences.*

Other Analog Strategies. Symbologies can be constructed using operations other than addition to produce numerals. Numerals can be related visually with their multiples or exponents (see Figure 4). The "arqam" system, illustrated in a later part of this article, produces a more or less arbitrary symbol for a transdecimal prime number (transdecimal meaning a numeral representing a quantity larger than the decimal single-digit numerals), modifying these in a way such that all of the numeral's multiples bear some distinguishing element from the basis prime numeral. Shaun Ferguson's system, illustrated in the *Dozenal Newscast*, Year 2, № 1, page 10; at first blush appears to be a system of purely creative symbols. There is a relationship among the numerals {2, 4, 8}, the exponents of 2, and somehow digit-ten. However, no similar relationship appears to be shared between the multiples or exponents of three [4]:



Indexed Value Strategy. The 7 segment LCD/LED readout plays an important role in Bill Schumacher's hexadecimal symbology, described in Vol. 33 № 3 page ↊. The numerals are read clockwise, representing bits, starting from the bottom right element. This element represents the unit bit. The bottom left element represents the presence of two when activated; the top left element four, and the top right element eight. One simply totals up the bits to read the numeral. The Schumacher numerals were expanded by Prof. Gene Zirkel to encode five dozen four digits. The Schumacher symbology and its Zirkel extension (illustrated later in this article) employ an indexing strategy wherein elements are assigned values, then these values are totalled to construct the overall value of the digit:

George P. Jelliss posted an entry in the DozensOnline internet forum describing a set of numerals which function much like the Schumacher / Zirkel symbologies. The Jelliss separate identity symbology takes advantage of the 7 segment LCD/LED readout to supply a matrix of elements which can be assigned indexed values. The two leftmost segments serve as a placeholder in all digits, effectively representing a value of zero. The middle horizontal segment has a value of 1, the top and bottom horizontals 2. When the two rightmost segments are activated, a value of 6 is added to the digit[5]:

Modular Symmetry Strategy. Rebuilding numerals for each dozenal digit enables one to take advantage of the high divisibility of the dozen and express the many modular relationships among twelve's digits. Decimal numerals might take advantage only of the relationship between 2 and 5, but dozenal numerals may be crafted to express the sequences {0, 2, 4, 6, 8, ↊}, {0, 3, 6, 9}, {0, 4, 8}, and {0, 6} in a manner that accentuates the fact, for example, when you see a 3, it's ¼ of a dozen. J. Halcro Johnson's reverse notation numerals possess a symmetry which expresses relationships between the thirds {0, 4, ƀ}: any numeral involving the form "4", whether erect or reversed, signifies an exact third of a dozen. Odd quarters are likewise represented in the sequence {0, 3, 6, ↋} by the "3" form, and the half is represented by "6".

My own 2003 "acýlin" or cyclical dozenal symbology repurposes the Hindu-Arabic numbers to fit the cycle of dozenal digits. In this system, the sequence {0 3 6 9} appears as {0 3 8 ↋}, so all quarters or multiples of three appear as some manifestation of the "three" base form. The sequence {0 4 8} appears as {0 ʒ ↋}, so all thirds or multiples of 4 use the "3"

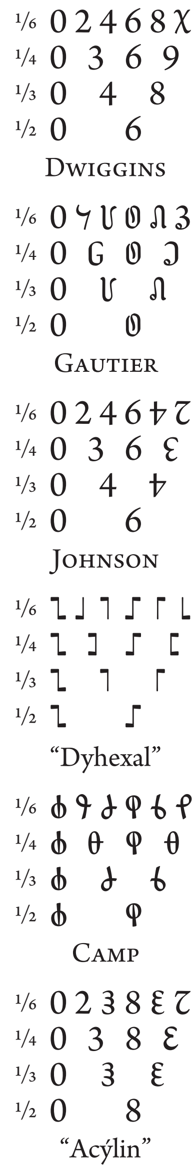


Figure 6. *Studies of various modular symmetries of some separate identity symbologies. The rows of each study lay out digits divisible by 2, 3, 4, or 6.*

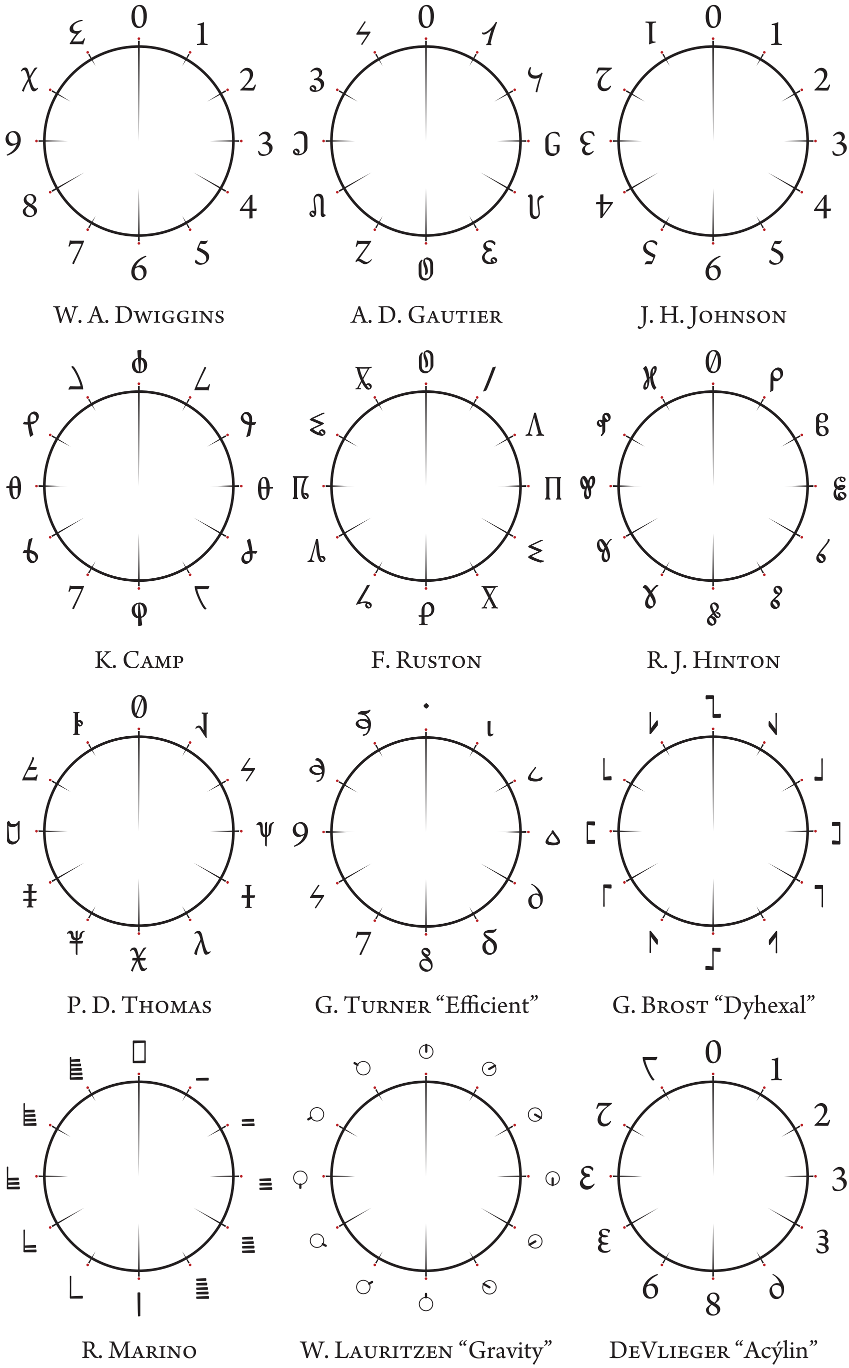


Figure 7. *Cyclical studies of several separate identity symbologies.*

base form. The symbol "8" is repositioned, because of its symmetry and likeness to a "twisted zero", to signify six. The concern here is appearance, although the set could be used to represent reverse notation as well.

0 1 2 3 Ʒ ∂ 8 6 ɛ Ɛ Շ ⌐

A. D. Gautier's symbology of 1858, published in the *Dozenal Newscast*, Year 2, № 1, page 10; accentuates dozenal halves, quarters, and thirds. Gautier seems to have derived his numeral 6 (0) by splitting the zero in half. The odd quarters {3, 9}, represented by {G, Ɔ} are either half of the zero, with minor distinguishing marks added, perhaps, to reduce their confusion. The thirds {4, 8} are represented by {Ʊ, ᔑ}, more or less a "v" or "u" shape pointing downward for 4 and upward for 8.

0 ⁄ ϟ G Ʊ Ƹ 0 Z ᔑ Ɔ 3 ϟ

Gerard Robert Brost's "dyhexal" symbology, which appears in Vol. 35; № 2 page Ҳ;, uses numerals which function as analogs which might be read like hands on a clock to literally indicate which digit the symbol represents. He states, "Identification is aided because when the numbers are arranged in order they form a symmetric pattern". Later he adds "All numbers that are evenly divisible by two but not three (i.e. two, four, eight, and dec) have one horizontal feature in one of four positions (˩ ˥ ┌ └)." Thus Brost's symmetrical symbology is devised to aid the user in adapting to and interpreting the new symbols. Due to Brost's conscious inclination to produce a symmetrical set of symbols, his system automatically accentuates the modular relationships among the dozenal digits. The halves {0, 6} are represented by {ƪ, ʃ}, odd quarters by {ב, ⊏}, thirds by {˥, ┌}, and the sequence of sixths, {0, 2, 4, 6, 8, Χ} by {ƪ, ˩, ˥, ʃ, ┌, └}. Additionally, relatively prime digits {1, 5, 7, Ɛ} are characterized by numerals with slanting "flag" strokes {ᛀ, ᛏ, ᚨ, ᛒ}:

ƪ ᛀ ˩ ב ˥ ᛏ ʃ ᚨ ┌ ⊏ └ ᛒ

Kingsland Camp described more than one "ideal" set of numerals in his article "Number Symbols" in Vol. 2 № 1 page 16;. These he contrasted with the Least Change or "transitional" symbologies, including the DSA's classic Dwiggins numbers. The symmetrical geometries of his numerals were intended to more purely convey the digit they represent, by their rotational orientations. Halves are represented by {ȸ, φ}, circles with vertical strokes through their centers, odd quarters by {θ, θ}, circles with strokes left or right. The numerals representing the range of quarters {0, 3, 6, 9} can be read like clock faces: {ȸ, θ, φ, θ}. Camp's notation expresses thirds by {ძ, ℎ}, and the sequence of sixths by {ȸ, Ϡ, ძ, φ, ℎ, ϙ}. Thus any numeral representing a sixth of a dozen not already covered by the quarters appears as a "fish" shaped symbol. Camp's notation also treats the relatively prime digits {1, 5, 7, Ɛ} in a way that accentuates them, a sharp angular numeral opening up to the numeral's position on a standard clock face: {∠, ⌐, 7, ⊿}:

ȸ ∠ Ϡ θ ძ ⌐ φ 7 ℎ θ ϙ ⊿

Application to Subdecimal Bases. We've considered dozenal and some hexadecimal proposals in this article and in Vol. 4Χ; № 2. Symbologies are not necessarily limited to these bases. Arguably the solution for "subdecimal" bases (those

Figure 8. *The first six dozen arqam.*

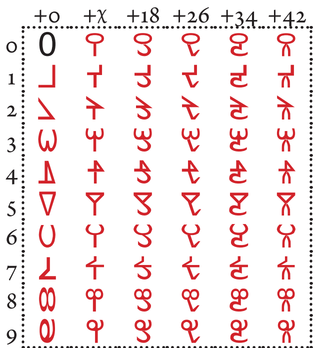


Fig 9. *Smith's sexagesimal numerals*

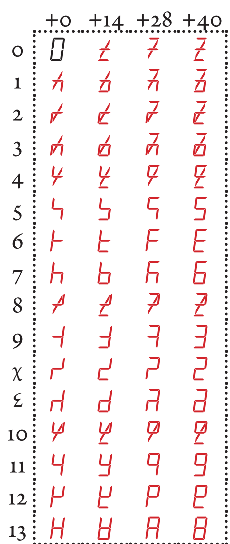


Fig χ. *Zirkel's expansion of Schumacher numerals*

smaller than ten) is simply to use the requisite portion of the Hindu-Arabic numeral set, ignoring the rest. This is what is encountered in practice, concerning octal and binary. In theory, one could devise other solutions for subdecimal symbologies. The binary used on NASA's Voyager Golden Record (see http://voyager.jpl.nasa.gov/spacecraft/images/VgrCover.jpg) uses a horizontal line to signify zero, a vertical line to signify one. This is a simple and pure rotational strategy:

An octal proposal which functions much like Schumacher's hexadecimal proposal can be seen at http://www.octomatics.org/. This symbology uses a pure exponential indexed value strategy, similar to the Schumacher and Zirkel hexadecimal and base five dozen four symbologies:

Exceedingly Transdecimal Bases. When we are working with "human scale" number bases, between about 6 or 7 and around one dozen or one dozen four, it's not too difficult to devise sufficient numeral symbols. The challenge increases in proportion to the size of the number base. When the base under consideration is several multiples of ten, the opposed principles begin to resemble one another. This is because the retention of the ten Hindu-Arabic symbols represents an increasingly smaller subset of the total number of necessary numerals.

Least Change. I produced and use a series of transdecimal digits called "arqam", Arabic for "numbers", which at first covered hexadecimal, then was extended to sexagesimal and beyond. Currently there are over two and a half gross total contiguous symbols. The digits zero through four dozen eleven appear below. This is a "creative" least change proposal writ large. The system uses a multiplicative strategy to generate digits (see Figure 5). The multiples of one dozen five begin { }, of one dozen seven { }, and extend themselves ever higher using basic graphic elements called "radicals". A small tight circle, normally at the foot of a base form, tends to double the value of that base form, as seen in two dozen ten ([illegible]) compared to one dozen five ([illegible]). A three-pointed wavelike radical at the top of a glyph normally means the base form is tripled: as seen in one dozen nine ([illegible]), compared to seven (7). There are radicals for relatively large numbers like one dozen four (seven times 14; = 7 × ρ = ϩ, ten times 14; = Ꞇ × ρ = ᘔ) and even three dozen nine (three times 39; = 3 × ℓ = ℛ, six times 39; = 6 × ℓ = ᶗ), as ever larger symbols are created. The symbols at first resemble plausible Hindu-Arabic-like numerals, then devanāgarī letters divorced from their top line, then Mandarin characters. The largest symbols in the set, which represent powers of prime numbers, use an exponential strategy similar to Ferguson in Figure 4 (powers of three: 1, 3, 9, ᒿ, ᕐ, ᕑ). See Figure 8 for the first six dozen of the "arqam" numerals.

Separate Identity. A rationalized, derivative Separate Identity sexagesimal symbology invented by Jean-Michel Smith appears at http://autonomyseries.com/Canon/Sexagesimal/. This symbology uses the "six on ten" or decimal "sub-base" arrangement also used by Babylonian cuneiform numerals, but applies this to Hindu-Arabic numerals. The numerals zero through nine are reinterpreted, rotated on their sides, and serve as a basic run sequence. Smith appends diacriticals to the bottom of the basic run sequence to shift the value of these ten basic glyphs up by multiples of ten. An example of this diacritical is the vertical line extending down from the "body" of the glyph, to add ten to the integer the body represents. This results in a relatively transparent "transitionary" system (to borrow the term from Kingsland Camp). Smith's numeral set is perhaps a direct conceptual descendant of Babylonian cuneiform numerals. (These symbols were created and produced by Jean-Michel Smith and appear in Figure 9 via the GNU license which is posted on his site.)

Prof. Gene Zirkel expanded Bill Schumacher's notation in Vol. 49; № 1 page 15;, extending the digits to represent the base of the sixth power of two. The kernel notion of this symbology was expressed in Vol. 33; № 3 page 11;. The bottommost horizontal element, when activated, represents the presence of one dozen four; while the topmost horizontal element represents the presence of two dozen eight. Zirkel's figures add a "slash" in the digits which preserves the integrity of the digits. See Figure χ.

We have reached the end of our tour through dozenal symbologies and beyond, and some of the tools and strategies one can use to design them. This does not rule out further possibilities, nor does it mean that the classification or strategies presented here are the only valid ones. Again, this framework and the strategies mentioned herein are intended merely to serve as tools. We look forward to your own creations, or comments on the numeral sets shown here.

Notes:

1 Retrievable at time of publishing at www.dozenalsociety.org.uk/basicstuff/hammond.htm, part of the official website of the Dozenal Society of Great Britain.

2 Retrieved in early 2009, originally at www.geocities.com/nigellus_albus/whittenswords/measure/dozchar.htm, currently at www.angelfire.com/whittenwords/measure/dozchar.htm. Mr. Whitten recently devised a third generation of his numerals.

3 The dot patterns displayed above Hinton's numerals do not precisely correspond to his original article.

4 This set of Ferguson numerals appears as Mr. Ferguson conveyed through personal correspondence in early December 2009.

5 Retrievable from the DozensOnline internet forum. George P. Jelliss (2005). Symbols for TEN and ELEVEN? (Internet forum thread in the "On Topic" Forum, "Number Bases" topic) http://z13.invisionfree.com/DozensOnline/index.php?showtopic=11, entry by user "GPJ" at 10:54 am 14 August 2005.